4. If you dig a hole right through the Earth and jump into it, describe your motion.

Suggested answer

We need to make a number of assumptions in order to simplify and model this question.

If we assume that there is no friction or other losses to dissipation (e.g. air resistance as we fall down the hole), and ignore the fact that the Earth's density is non-uniform, as well as neglecting forces that acts on the person due to the Earth's spin we can simplify our problem significantly.

As you move down the hole towards the centre of the Earth, the force acting on you will reduce linearly:

$$F = -\frac{GMm}{r^2} = -\frac{G(\frac{4}{3}\pi r^3 \rho)m}{r^2} = -\frac{4}{3}G\pi\rho rm = -kr$$

$$a = \frac{F}{m} = -\frac{4}{3}G\pi\rho r$$

Where '*m*' is your mass, '*M*' is the mass of the Earth within a sphere radius '*r*', '*r*' is the distance from the centre of the Earth, and ρ is the density of the Earth. '*k*' is a constant equal to 4/3($G\pi\rho m$) and defined by putting all of the constants together in order to simplify the expression.

The force is directly proportional to the displacement from a reference point (in this case the displacement from the centre of the Earth) and acts in the opposite direction to the motion. This is the characteristic of Simple Harmonic Motion. Therefore if you were to jump into this well given the assumptions stated, you would exhibit SHM. Your acceleration would decrease as you approach the centre of the Earth, where it would decrease to zero.

Your initial acceleration would be the surface acceleration of gravity:

$$g = \frac{GM_{Earth}}{r_{Earth}^2} = 9.8ms^{-2}$$

Using this, we can simplify the first expression:

$$\frac{4}{3}G\pi\rho r_{Earth} = g$$